$\ell_\perp$ to ℓ, respectively. By the Triangle Inequality, we have

$$\sum_{i=1}^{4n}|x_i| + \sum_{i=1}^{4n}|y_i| \ \geq\ \sum_{i=1}^{4n}|s_i| \ =\ 4n\,.$$

We consider three cases. First, if $\sum\limits_{i=1}^{4n}|x_i| > 2n$, then there is a point P on ℓ that belongs to two of the x_i. In this case, the perpendicular to ℓ through P is a suitable choice for ℓ'. Second, if $\sum\limits_{i=1}^{4n}|y_i| > 2n$, then there is a point Q on $\ell_\perp$ that belongs to two of the y_i. The parallel line to ℓ through Q is a suitable choice for ℓ'. It remains to consider the case $\sum\limits_{i=1}^{4n}|x_i| = \sum\limits_{i=1}^{4n}|y_i| = 2n$. In this situation, the parallel line to ℓ through the midpoint of Γ is adequate for ℓ'.

3. Let a, b, c, and d be positive real numbers such that $a+b+c+d=1$. Prove that $6\big(a^3+b^3+c^3+d^3\big) \geq \big(a^2+b^2+c^2+d^2\big)+\frac{1}{8}$.

Solved by Mohammed Aassila, Strasbourg, France; Arkady Alt, San Jose, CA, USA; and Titu Zvonaru, Comănești, Romania. We give Alt's presentation.

By the Power Mean Inequality

$$\begin{aligned}\frac{a^3+b^3+c^3+d^3}{4} &\geq \left(\frac{a+b+c+d}{4}\right)^3;\\ a^3+b^3+c^3+d^3 &\geq \frac{(a+b+c+d)^3}{16} = \frac{1}{16}\,,\end{aligned}$$

and by Chebychev's inequality

$$a^3+b^3+c^3+d^3 \ \geq\ \frac{a+b+c+d}{4}(a^2+b^2+c^2+d^2) \ =\ \frac{a^2+b^2+c^2+d^2}{4}\,.$$

This yields

$$\begin{aligned}&6(a^3+b^3+c^3+d^3)\\ &\quad= 4(a^3+b^3+c^3+d^3)+2(a^3+b^3+c^3+d^3)\\ &\quad\geq (a^2+b^2+c^2+d^2)+\frac{1}{8}\,,\end{aligned}$$

as desired

Next we move to solutions to problems of the Hong Kong Team Selection Test 1 given at [2009 : 214].

1. Find the integer solutions of the equation $7(x+y)=3\big(x^2-xy+y^2\big)$.